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Non-BPS Dp-brane in Dk-brane background

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Abstract: In this paper we will study the dynamics of a non-BPS Dp-brane in the background of N BPS Dk-branes.

Keywords: D-branes, Tachyon Condensation.

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Contents

1. Introduction

One of the most important challenges in string theory is to formulate the framework for addressing the string dynamics in time dependent backgrounds. Due to the fundamental work by A. Sen [1, 2, 3] there has been significant progress in the understanding time-dependent phenomena in open string theory:[1] decay of unstable D-brane or brane-antibrane pair can be described in terms of condensation of open string tachyon.

Another problem where time dependent string dynamics is involved is situation when a probe Dp-brane moves in the background of N NS5-branes. Very interesting analysis of this process was given by Kutasov in [5].[2] It was shown there that the dynamics of the radial mode of the BPS D-brane resembles the tachyon rolling dynamics of unstable D-brane. In particular, for appropriate background of NS5-branes the radion effective action takes exactly the same functional form as the tachyon effective action for unstable D-brane and proposed to view the radion rolling dynamics as a sort of "geometrical" realisation of tachyon rolling dynamics on unstable D-brane.

An important message from the study of the real time open string dynamics is that the tachyon effective action Dirac-Born-Infeld (DBI)-like type [19]–[22] captures many aspects of rolling tachyon solution of string theory. Some suggestions why the tachyon DBI action is such remarkable successful in the description of the tachyon dynamics were presented in [23, 25, 26].[3] Then it is certainly useful to gain as many informations from the study of the effective field theory description of various D-brane configurations as possible. For that reason we have recently analysed the effective field theory description of the non-BPS Dp-brane in the background of N NS5-branes [10, 11]. We have shown that the tachyon effective action can be written in the form that suggests geometric origin of the tachyon

[1] For recent review, see [4] where extensive list of relevant papers can be found.

[2] The extension of this approach can be found in [6]–[18].

[3] For alternative point of view on the problem of tachyon effective action, see [27].

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as the new embedding coordinate. Then the tachyon effective action can be considered as an action for a BPS D-brane moving in nontrivial eleven dimensional background. This observation supports the ideas that were previously presented in [5, 13]. Then we have studied the dynamics of the non-BPS Dp-brane in the NS5-brane background. We have considered two cases: The first one when the tachyon and radial mode were time dependent. We have argued that the general analysis of this problem is very difficult thanks to the non-existence of the additional conservation charge related to the presence of the tachyon. On the other hand in the region of the field theory space where T is large and unstable D-brane was close to the NS5-branes the tachyon effective action posses additional symmetry leading to the emergence of the new conserved charge. Using this fact we were able to determine the time dependence of the tachyon and radial mode in this particular region of the field theory space. The second example of the tachyon condensation that we have studied was the situation when the tachyon was function of one single spatial coordinate on the worldvolume of unstable D-brane while the embedding modes were time dependent.

In this paper we extend the analysis performed in [10] to the case of the motion of a non-BPS Dp-brane in the background of N BPS Dk-branes. The situation when a BPS Dp-brane moves in the Dk-brane background was studied previously in [12, 16] and in [31]. Even if the analysis of this problem is slightly more complicated than the study of the dynamics of a BPS Dp-brane in the NS5-brane background one can still find an analogue between the rolling radion and rolling tachyon. In particular, it was shown in [16] that the effective field theory description of the dynamics of the BPS Dp-brane in the background of N Dk-branes can be still mapped to the tachyon like effective action for unstable Dp-brane. Then it is natural to extend this analysis to the case when a non-BPS Dp-brane moves in the background of N Dk-branes.

More precisely, in the next section (2) we will write the general form of the non-BPS Dp-brane effective action in the background of N BPS Dk-branes. We will show that the tachyon mode has striking similarity with additional embedding coordinate. Then we will focus on the study of the dynamics of the non-BPS Dp-brane in the Dk-brane background. We will firstly consider the case of the time dependent tachyon and time dependent radial mode. After the general discussion of the properties of the non-BPS Dp-brane effective action for this dynamical situation we will consider in section (3) the case when the tachyon is large and unstable Dp-brane is near the worldvolume of the coincident N Dk-branes. Following [10] we will try to find an additional symmetry of the tachyon effective action. Since the form of the metric and dilaton produced by N Dk-branes is slightly more complicated than in the case of the NS5-brane background we will need to consider more general form of the transformation of the tachyon and the radial mode. However the condition that probe Dp-brane action has to be invariant under these transformations further restrict the dimension of the background Dk-branes, namely we will see that the spatial dimension of Dk-branes should be equal to three.

Then for the case of the background of N D3-branes we will be able to determine the time dependence of the tachyon and radial mode. We will also find that the time dependence of the radial mode and the tachyon depends on the spatial dimension of the worldvolume of unstable Dp-brane.

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As the second example of the tachyon dynamics we will consider in section (4) the case when the tachyon is spatial dependent and the radial mode is time dependent. We will see that the tachyon condensation results to the emergence of a singular tachyon kink that has natural interpretation as a D$(p-1)$-brane that moves in the background of N Dk-branes. Then we will argue that the description of the dynamics of the resulting configuration can be performed in the same way as in [31, 16].

Finally, in conclusion (5) we outline our result and suggest possible extension of this work.

2. The effective action for non-BPS Dp-brane in Dk-brane background

In this section we will analyse the motion of a non-BPS Dp-brane in the stack of coincident and static Dk-branes using the tachyon effective action proposed in [19, 21, 22, 20]. The metric, the dilaton (Φ), and the R-R field (C) for a system of N coincident Dk-branes is given by

$$g_{\alpha\beta} = H_k^{-\frac{1}{2}}\eta_{\alpha\beta}\,, \qquad g_{mn} = H_k^{\frac{1}{2}}\delta_{mn}\,, \qquad (\alpha,\beta = 0,1,\dots,k,\ m,n = k+1,\dots,9)\,,$$
$$e^{2\Phi} = H_k^{\frac{3-k}{2}}\,, \qquad C_{0\dots k} = H_k^{-1}\,, \qquad H_k = 1 + \frac{Ng_s}{r^{7-k}}\,, \tag{2.1}$$

where H_k is a harmonic function of N Dk-branes satisfying the Green function equation in the transverse space.

Now let us consider a non BPS Dp-brane with $p < k$ that is inserted in the background (2.1) with its spatial section stretched in directions $(x^1,\dots,x^p)$. We will label the worldvolume of the non-BPS Dp-brane by $\xi^\mu, \mu = 0,\dots,p$ and use reparametrisation invariance of the worldvolume of the Dp-brane to set $\xi^\mu = x^\mu$. The position of the D-brane in the transverse directions $(x^{p+1},\dots,x^9)$ gives to rise to scalar fields on the worldvolume of D-brane, $(X^{p+1}(\xi^\mu),\dots X^9(\xi^\mu))$. According to [19, 21, 22, 20] the non-BPS Dp-brane (DBI)-like effective action takes the form

$$S = -\tau_p \int d^{p+1}\xi V(T) e^{-\Phi-\Phi_0}\sqrt{-\det(G_{\mu\nu} + B_{\mu\nu} + \partial_\mu T\partial_\nu T)} + S_{WZ}\,, \tag{2.2}$$

where $\tau_p = \frac{\sqrt{2}}{(2\pi)^{p+1}}$ is a tension of the non-BPS Dp-brane[4] and where $V(T)$ is a tachyon potential. According to papers [24, 23] we presume that it takes the form

$$V(T) = \frac{1}{\cosh\frac{T}{\sqrt{2}}}\,. \tag{2.3}$$

Finally, in (2.2) we have also included the coupling of the non-BPS Dp-brane to the Ramond-Ramond field that is governed by the Wess-Zumino term [29, 28]

$$S_{WZ} = \frac{1}{\sqrt{2\pi}}\int V(T)dT \wedge C_{RR}\,. \tag{2.4}$$

[4]We work in units where $\alpha' = 1$.

Even if the action (2.2) presented above is the most familiar one that is used for the description of a unstable D-brane we will prefer to describe the dynamics of the non-BPS Dp-brane using the tachyon effective action proposed in [23] that in the flat spacetime takes the form

$$\begin{aligned} S &= -\tau_p \int d^{p+1}\xi e^{-\Phi_0} \frac{1}{\sqrt{1+\frac{T^2}{2}}} \sqrt{-\det\left(\eta_{\mu\nu} + (1+\frac{T^2}{2})^{-1}\partial_\mu T \partial_\nu T\right)} + S_{WZ} \\ &= -\tau_p \int d^{p+1}\xi \sqrt{F} e^{-\Phi_0} \sqrt{-\det(\eta_{\mu\nu} + F\partial_\mu T\partial_\nu T)} + S_{WZ}\,, \\ F &= \frac{1}{1+\frac{T^2}{2}}\,. \end{aligned} \tag{2.5}$$

It can be shown [23] that there exists field redefinition that maps (2.5) to (2.2). The existence of this field redefinition implies that these two actions are equivalent, at least at the flat spacetime.

As a next step we will presume that the action (2.5) correctly describes the dynamics of the non-BPS Dp-brane in the general closed string background where the action (2.5) takes the form

$$S = -\tau_p \int d^{p+1}\xi e^{-\Phi} \sqrt{F} \sqrt{-\det(G_{\mu\nu} + F\partial_\mu T \partial_\nu T)} + S_{WZ}\,, \tag{2.6}$$

where the determinant in (2.2) runs over the worldvolume directions $\mu = 0,\dots,p$, $G_{\mu\nu}$ and $B_{\mu\nu}$ are induced metric and B field on the non-BPS Dp-brane

$$\begin{aligned} G_{\mu\nu} &= \frac{\partial X^A}{\partial \xi^\mu}\frac{\partial X^B}{\partial \xi^\nu} G_{AB}(X)\,, \\ B_{\mu\nu} &= \frac{\partial X^A}{\partial \xi^\mu}\frac{\partial X^B}{\partial \xi^\nu} B_{AB}(X)\,. \end{aligned} \tag{2.7}$$

In this notation the indices $A, B = 0,\dots,9$ run over the whole ten dimensional spacetime so that G_{AB} and B_{AB} are metric and B-field in ten dimensions. It is however important to stress that the step from (2.5) to (2.6) is nontrivial since it is well known that the form of the tachyon effective action strongly depends on the field theory space where it is presumed to be valid [27].

Let us now consider the non-BPS Dp-brane in the background of N Dk-branes given in (2.1) where the worldvolume of the unstable Dp-brane is stretched along the worldvolume of Dk-branes. If we choose the static gauge $\xi^\mu = x^\mu,\ \mu = 0, 1,\dots,p$ then the action (2.6) takes the form

$$S = -\tau_p \int d^{p+1}\xi \sqrt{F} H_k^{\frac{k-3}{4}} \sqrt{-\det(G_{\mu\nu} + F\partial_\mu T\partial_\nu T)} + S_{WZ}\,, \tag{2.8}$$

where the induced metric is equal to

$$G_{\mu\nu} = H_k^{-\frac{1}{2}} \eta_{\mu\nu} + H_k^{-\frac{1}{2}} \partial_\mu Y^i \partial_\nu Y^i + \partial_\mu X^m \partial_\nu X^m H_k^{\frac{1}{2}}\,. \tag{2.9}$$

From here on we reserve the coordinates X^m with $(m = k+1,\dots,9)$ for dimensions transverse to Dk-brane and the coordinates Y^i with $(i = p+1,\dots,k)$ for those dimensions transverse to non-BPS Dp-brane, but parallel to the Dk-brane.

Looking at the form of the tachyon effective action (2.8) we can now think about it as the DBI action for Dp-brane embedded in eleven dimensional spacetime with the background metric and dilaton

$$ds^2 = H^{-\frac{1}{2}}\eta_{\alpha\beta}dx^\alpha dx^\beta + H^{\frac{1}{2}}\delta_{mn}dx^m dx^n + FdT^2\,, \qquad e^{2\Phi} = \frac{H_k^{\frac{3-k}{2}}}{F}\,. \tag{2.10}$$

Even if this is clearly very promising idea it remains to be seen whether it has more precise physical justification.

Now we will study the equation of motion that arises from (2.8). As the first example we will consider the case when the tachyon and the embedding fields Y^i, X^m are time dependent. In this case the matrix

$$\mathbf{A}_{\mu\nu} \equiv H_k^{-\frac{1}{2}}\eta_{\mu\nu} + H_k^{-\frac{1}{2}}\partial_\mu Y^i\partial_\nu Y^i + H_k^{\frac{1}{2}}\partial_\mu X^m\partial_\nu X^m + F\partial_\mu T\partial_\nu T \tag{2.11}$$

takes the form

$$\mathbf{A}_{\mu\nu} = \begin{pmatrix} -H_k^{-\frac{1}{2}} + H_k^{-\frac{1}{2}}\dot{Y}^i\dot{Y}^i + H_k^{\frac{1}{2}}\dot{X}^m\dot{X}^m + F\dot{T}^2 & 0 \\ 0 & H_k^{-\frac{1}{2}}\mathbf{I}_{p\times p} \end{pmatrix} \tag{2.12}$$

so that

$$\det\mathbf{A} = -H_k^{-\frac{1}{2}}(1 - \dot{Y}^i\dot{Y}^i - H_k\dot{X}^m\dot{X}^m - H_k^{\frac{1}{2}}F\dot{T}^2)H_k^{-\frac{p}{2}}\,. \tag{2.13}$$

Then the action (2.8) takes the form

$$S = -\tau_p V_p\int dt\sqrt{F}H_k^{\frac{k-p-4}{4}}\sqrt{1 - \dot{Y}^i\dot{Y}^i - H_k\dot{X}^m\dot{X}^m - H_k^{\frac{1}{2}}F\dot{T}^2} \equiv -\int dt\mathcal{L}\,. \tag{2.14}$$

Note that for $T = 0$ the action (2.14) agrees with the action studied in [31]. We can also see that for the time dependent tachyon the WZ term vanishes identically using the fact that $dT = \dot{T}dt$ and that the nonzero components of the RR fields are these with the factor $C_{01\dots p}$ so that the wedge product $dT\wedge C$ vanishes.

To proceed we now determine some conserved charges that follow from (2.14). Since the action does not explicitly depend on time the hamiltonian

$$H = P_i\dot{Y}^i + P_m\dot{X}^m + \Pi\dot{T} - \mathcal{L} = \frac{\tau_p V_p\sqrt{F}H_k^{\frac{k-p-4}{4}}}{\sqrt{1 - \dot{Y}^i\dot{Y}^i - H_k\dot{X}^m\dot{X}^m - H_k^{\frac{1}{2}}F\dot{T}^2}} \tag{2.15}$$

is conserved. In (2.15) the momenta conjugate to T, X^m and Y^i are equal to

$$P_m = \frac{\delta\mathcal{L}}{\delta\dot{X}^m} = \tau_p V_p H_k^{\frac{k-p-4}{4}}\frac{H_k\dot{X}^m}{\sqrt{1 - \dot{Y}^i\dot{Y}^i - H_k\dot{X}^m\dot{X}^m - H_k^{\frac{1}{2}}F\dot{T}^2}}\,,$$

$$P_T = \frac{\delta \mathcal{L}}{\delta \dot{T}} = \tau_p V_p H_k^{\frac{k-p-4}{4}} \frac{H^{\frac{1}{2}} F \dot{T}}{\sqrt{1 - \dot{Y}^i \dot{Y}^i - H_k \dot{X}^m \dot{X}^m - H_k^{\frac{1}{2}} F \dot{T}^2}} ,$$

$$P_i = \frac{\delta \mathcal{L}}{\delta \dot{Y}^i} = \tau_p V_p H_k^{\frac{k-p-4}{4}} \frac{\dot{Y}^i}{\sqrt{1 - \dot{Y}^i \dot{Y}^i - H_k \dot{X}^m \dot{X}^m - H_k^{\frac{1}{2}} F \dot{T}^2}} . \qquad (2.16)$$

The next conserved charge corresponds to the manifest rotation symmetry $SO(9-k)$ of the transverse R^{9-k} space. In fact, it easy to see that the action is invariant under the transformation

$$X'^m(t) = \Lambda^m_n X^n(t) , \qquad \Lambda^m_k \delta_{mn} \Lambda^n_l = \delta_{kl} . \qquad (2.17)$$

Using this symmetry we can restrict ourselves to the motion in the transverse (x^8, x^9) plane[5] where the corresponding conserved angular momentum is equal to

$$L \equiv L^9_8 = \tau_p V_p \sqrt{F} H_k^{\frac{k-p}{4}} \frac{\dot{X}^9 X^8 - \dot{X}^8 X^9}{\sqrt{1 - \dot{Y}^i \dot{Y}^i - H_k \dot{X}^m \dot{X}^m - H_k^{\frac{1}{2}} F \dot{T}^2}} = P^8 X^9 - P^9 X^8 . \qquad (2.18)$$

In terms of the polar coordinates defined as

$$X^8 = R \cos\theta , \qquad X^9 = R \sin\theta \qquad (2.19)$$

the conserved energy and angular momentum take the form

$$E = \tau_p V_p \sqrt{F} H_k^{\frac{k-p-4}{4}} \frac{1}{\sqrt{1 - \dot{Y}^i \dot{Y}^i - H_k(\dot{R}^2 + R^2 \dot{\theta}^2) - H_k^{\frac{1}{2}} F \dot{T}^2}} ,$$

$$L = \tau_p V_p \sqrt{F} H_k^{\frac{k-p}{4}} \frac{R^2 \dot{\theta}}{\sqrt{1 - \dot{Y}^i \dot{Y}^i - H_k(\dot{R}^2 + R^2 \dot{\theta}^2) - H_k^{\frac{1}{2}} F \dot{T}^2}} . \qquad (2.20)$$

It is also convenient to work with the densities so that we strip off the volume factor V_p in (2.20). Then in what follows we will consider E, L as corresponding densities.

Finally, since the action (2.14) does not explicitly depend on Y^i it is clear that the momenta P_i are conserved as well. In order to simplify expressions that we will obtain below we will consider the case when all momenta P_i vanish and hence $\dot{Y}^i = 0$.

As can be seen from the form of the tachyon effective action (2.14) and corresponding conserved charges it is very difficult to solve the resulting differential equations for R, T and θ in the full generality. Moreover, the fact that there is missing the third conserved charge makes the analysis even worse. On the other hand we have shown in [10] that for the case of the motion of a non-BPS Dp-brane in the background of N NS5-branes one can find the region in the field theory space spanned by T, R, where new additional symmetry of the non-BPS Dp-brane action emerges. The existence of this symmetry allows to define new conserved charge and one can explicitly describe the dynamics of the non-BPS Dp-brane. For that reason it would be desirable to find such a symmetry for non-BPS Dp-brane in the Dk-brane background as well.

[5] This restriction also implies that the background Dk-branes are those with $k < 8$.

Before we address this question we would like to say few words about the dynamics of non-BPS Dp-brane in the Dk-background, following [30, 31]. For convenience we again write the action that governs the dynamics of a non-BPS Dp-brane in the (x^8, x^9) plane

$$S = -\tau_p V_p \int dt H_k^{\frac{k-p-4}{4}} \sqrt{F} \sqrt{1 - H_k(\dot{R}^2 + R^2\dot{\theta}^2) - F H_k^{\frac{1}{2}} F \dot{T}^2} \,. \tag{2.21}$$

From this action we immediately obtain the momenta P_R, P_T and P_θ

$$\begin{aligned} P_R &= \tau_p V_p dt H_k^{\frac{k-p-4}{4}} \sqrt{F} \frac{H_k \dot{R}}{\sqrt{1 - H_k(\dot{R}^2 + R^2\dot{\theta}^2) - F H_k^{\frac{1}{2}} F \dot{T}^2}} \,, \\ P_\theta &= \tau_p V_p H_k^{\frac{k-p-4}{4}} \sqrt{F} \frac{R^2 H_k \dot{\theta}}{\sqrt{1 - H_k(\dot{R}^2 + R^2\dot{\theta}^2) - F H_k^{\frac{1}{2}} F \dot{T}^2}} \,, \\ P_T &= \tau_p V_p H_k^{\frac{k-p-4}{4}} \sqrt{F} \frac{F H_k^{\frac{1}{2}} \dot{T}}{\sqrt{1 - H_k(\dot{R}^2 + R^2\dot{\theta}^2) - F H_k^{\frac{1}{2}} F \dot{T}^2}} \,. \end{aligned} \tag{2.22}$$

Hence the hamiltonian density is equal to

$$H = \frac{1}{V_p}\left(P_R \dot{R} + P_\theta \dot{\theta} + P_T \dot{T} - \mathcal{L}\right) = \sqrt{\tau_p^2 H_k^{\frac{k-p-4}{2}} F + \frac{P_R^2}{H_k} + \frac{P_\theta^2}{R^2 H_k} + \frac{P_T^2}{F H_k^{\frac{1}{2}}}} \,. \tag{2.23}$$

Since the hamiltonian is monotonically increasing function of canonical momenta it is bounded from bellow by the potential

$$V(T, R) \equiv H(P_R = P_T = P_\theta = 0) = \tau_p H_k^{\frac{k-p-4}{4}} \sqrt{F} \,. \tag{2.24}$$

For fixed T this potential is attractive for $k - p < 4$ and repulsive when $k - p > 4$.

We also see that the lagrangian in (2.21) does not explicitly depend on θ and consequently the momentum P_θ is conserved. This fact allows us to introduce the effective potential for T, R that has the form

$$V_{\text{eff}}(T, R) \equiv H(P_R = P_T = 0) = \sqrt{\tau_p^2 H_k^{\frac{k-p-4}{2}} F + \frac{P_\theta^2}{H_k R^2}} \,. \tag{2.25}$$

The utility of this potential follows from the fact that $\partial E / \partial P_{R,T} > 0$. Then the allowed ranges of R, T can be found by standard device by plotting V_{eff} against T, R and finding those T, R where $E > V_{\text{eff}}(R, T)$.

In fact, our goal is to find the stable orbits corresponding to the extremum of the potential. The possible extrema of V_{eff} with respect to T follow from the equation

$$0 = \frac{\delta V_{\text{eff}}(T, R)}{\delta T} = -\frac{\tau_p H_k^{\frac{k-p-4}{2}}}{2\sqrt{\tau_p^2 H_k^{\frac{k-p-4}{2}} F + \frac{P_\theta^2}{H_k R^2}}} \frac{T}{(1 + \frac{T^2}{2})^2} \,. \tag{2.26}$$

The first extremum corresponds to $T = 0$ with the standard interpretation as the unperturbed unstable Dp-brane. The analysis of the dynamics of non-Dp-brane sitting at its unstable point $T = 0$ is similar to the analysis performed in [30, 31] as we will review bellow.

As can be seen from (2.26) the stable extrema of the effective potential occur at $T = \pm\infty$. It is believed that these extrema correspond to some form of the pressureless gas. It would be certainly interesting to study the dynamics of this mysterious form of matter at the vicinity of the Dk-branes and we hope to return to this problem in future.

Let us now consider the case when the tachyon sits in its unstable minimum $T = 0$ and study the properties of the effective potential (2.25) for small and large R. For $R \to 0$ the effective potential (2.25) is equal to

$$V_{\text{eff}}(R, T = 0) = \sqrt{\tau_p^2 (Ng_s)^{\frac{(k-p-4)}{2}} R^{\frac{(k-7)(k-p-4)}{2}} + \frac{P_\theta^2 R^{5-k}}{Ng_s}} . \tag{2.27}$$

Since we presume that $k < 7$ the sign of the first exponent in (2.27) is given by sign of $k - p - 4$. Then we can find for small R following limits:

- For $k = 6$ the last term in (2.27) diverges ensuring that the potential diverges at origin.
- For $k = 5$ the last term approaches the constant for small r. Then for $p = 4, 2$ the potential approaches the constant $\frac{P_\theta}{\sqrt{Ng_s}}$ in this limit. On the other hand for $p = 0$ the potential diverges as $V_{\text{eff}} \sim \frac{1}{\sqrt{R}}$.
- For $k = 4$ the last term vanishes at the origin and the potential converges to zero since the first term scales as R^{3p} for $p = 3, 1$.
- For $k = 3$ the last term again vanishes at the origin. The first term scales as $R^{2(p+1)}$ that converges to zero for $p = 0, 2$.
- For $k = 2$ the last term again vanishes at the origin and the potential vanishes since the first term scales as $R^{\frac{5(2+p)}{2}}$.
- Finally, for $k = 1$ the last term vanishes and the first term scales as $R^{p(p+3)}$.

On the other hand, in the large R limit we instead find

$$V_{\text{eff}}(R, T = 0) = \sqrt{\tau_p^2 \left(1 + \frac{(k-p-4)Ng_s}{2R^{7-k}}\right) + \frac{P_\theta^2}{R^2}} \tag{2.28}$$

and we see that this potential approaches the constant for $R \to \infty$. How this limit is approaching depends on p, k in the following ways:

- For $k = 6$ the second term dominates and the potential approaches its limit from bellow.

- For $k=5$ the way the potential approaches its limiting value depends on p. For $p=0$ we have $\frac{\tau_p^2 N g_s(k-p-4)}{2}+P_\theta^2=\frac{\tau_p^2 N g_s}{2}+P_\theta^2>0$ and the potential is reached from above. On the other hand for $p=2,4$ the way the potential approaches the limiting value depends on the sign of $\frac{\tau_p^2 N g_s(k-p-4)}{2}+P_\theta^2=\frac{\tau_p^2 N g_s(1-p)}{2}+P_\theta^2$.
- For $k<5$ the last term always dominates and the potential approaches its limit from above.

Combining the behaviour for small and large R we obtain following picture:

- For $k=6$ the V_{eff} reaches the minimum away from $R=0$ and bound orbit exist. The analysis of this situation is the same as in the case of BPS Dp-brane studied in [31]. It is also clear that these bound orbits are unstable since the tachyon is sitting in its unstable maximum $T=0$. Note also that as opposite to the case of BPS Dp-brane there cannot exist the supersymmetric state characterised by condition $k-p=4$ since for unstable Dp-brane probe $k-p$ is always odd number.
- If $k=5$ and $p=2,4$ the existence of minimum depends on P_θ and there is a bound orbit if $\frac{\tau_p^2 N g_s(p-1)}{2}<P_\theta^2$. For $p=0$ there are not bound orbits.
- For $k<5$ the potential has the minimum at the origin that is smaller than at infinity. This implies that localised orbits exist.

Once again we must stress that the picture outlined above corresponds to the situations when $T=0$. On the other hand we will rather consider the case when the tachyon is dynamical as well. In fact, since the analysis of an unstable D-brane with the vanishing tachyon is almost the same as the analysis of the dynamics of BPS Dp-brane in Dk-brane background that was studied very carefully in [31] we will not discuss it here further.

3. Large T and small R

We have shown in our previous paper [11] where the dynamics of non-BPS Dp-brane in the background of NS5-branes was studied that for large value of tachyon and in the region close to the worldvolume of N NS5-branes a non-BPS D-brane action has additional symmetry that considerably simplifies the analysis of the time evolution of the non-BPS Dp-brane. Then it is natural to ask the question whether similar symmetry emerges in case when the non-BPS Dp-brane is embedded in the background of N Dk-branes in the region of the field theory space when $\frac{\lambda}{R^{7-k}}\gg 1,\ \lambda\equiv Ng_s\ ,\frac{T^2}{2}\gg 1$. In this region of the field theory space spanned by T,R the action (2.14) takes the form

$$S=-\tau_p\sqrt{2}V_p\lambda^{\frac{k-p-4}{4}}\int dt\frac{1}{TR^{\frac{(7-k)(k-p-4)}{4}}}\sqrt{1-\frac{\lambda}{R^{7-k}}(\dot{R}^2+R^2\dot{\theta}^2)-\frac{2\sqrt{\lambda}}{T^2R^{\frac{7-k}{2}}}\dot{T}^2}\,. \tag{3.1}$$

Let us now demand that the action (3.1) should be invariant under following transformations

$$t'=\Lambda^\alpha t\,,\qquad T'(t')=\Lambda^\beta T(t)\,,\qquad R'(t')=\Lambda^\gamma R(t)\,,\qquad \theta'(t')=\Lambda^\delta\theta(t)\,. \tag{3.2}$$

Now the requirement that the transformed and original action (3.1) should be equal leads to the set of following conditions

$$\begin{aligned}
\alpha-\beta-\frac{(7-k)(k-p-4)}{4}\gamma &= 0\,, \\
(k-7)\gamma+2\gamma-2\alpha &= 0 \Rightarrow \alpha=\frac{(k-5)\gamma}{2}\,, \\
(k-7)\gamma+2\gamma+2\delta-2\alpha &= 0 \Rightarrow \delta=0\,, \\
\frac{k-7}{2}\gamma-2\alpha &= 0 \Rightarrow (k-7)\gamma=4\alpha\,.
\end{aligned} \tag{3.3}$$

If we combine these equations then we obtain following result

$$\gamma=-\alpha \Rightarrow (k-3)\alpha=0 \tag{3.4}$$

with the solution that $k=3$ and α is arbitrary or $k\neq 3$ and $\alpha=0$. The second solution implies $\gamma=\delta=0$ and hence there is not any symmetry at all. More interesting is the first case. Then from the second equation in (3.3) we obtain

$$\alpha-\beta-\frac{(7-k)(k-p-4)}{4}\gamma=0 \Rightarrow \beta=-p\alpha\,, \tag{3.5}$$

where now α is arbitrary. In fact, different values of α correspond to different values of Λ since

$$t'=\Lambda^{\alpha}t=\tilde{\Lambda}t\,. \tag{3.6}$$

Then we can fix its value to be equal to an arbitrary number and we choose $\alpha=-1$. To recapitulate, the transformation under which the non-BPS Dp-brane action (3.1) in the background of N D3-branes

$$S=-\frac{\tau_p\sqrt{2}V_p}{\lambda^{\frac{p+1}{4}}}\int dt\frac{R^{p+1}}{T}\sqrt{1-\frac{\lambda}{R^4}(\dot{R}^2+R^2\dot{\theta}^2)-\frac{2\sqrt{\lambda}}{T^2R^2}\dot{T}^2}\equiv-\int dt\mathcal{L}\,. \tag{3.7}$$

is invariant takes the form

$$t'=\lambda^{-1}t\,,\qquad R'(t')=\lambda R(t)\,,\qquad \theta'(t')=\theta(t)\,,\qquad T'(t')=\lambda^{p}T(t)\,. \tag{3.8}$$

The conserved charge that generates these transformations takes the form

$$D=-tH-pTP_T-RP_R\,. \tag{3.9}$$

Now we are ready to study the time evolution of a non-BPS Dp-brane in the background of N D3-branes for large T and small R. It turns out that it is convenient to work in the hamiltonian formalism. The hamiltonian that follows from (3.7) takes the form

$$H=P_R\dot{R}+P_\theta\dot{\theta}+P_T\dot{T}-\mathcal{L}=\sqrt{\frac{2\tau_p^2R^{2(p+1)}}{\lambda^{\frac{p+1}{2}}T^2}+\frac{P_R^2R^4}{\lambda}+\frac{P_\theta^2R^2}{\lambda}+\frac{P_T^2T^2R^2}{2\sqrt{\lambda}}}\,. \tag{3.10}$$

Since the hamiltonian (3.10) does not explicitly depend on θ we get that P_θ is conserved

$$\dot{P}_\theta=-\frac{\partial H}{\delta\theta}=0\,. \tag{3.11}$$

On the other hand the time dependence of T and R is determined by the equations of motion that follow from (3.10)

$$\begin{aligned}
\dot{R} &= \frac{\partial H}{\partial P_R} = \frac{P_R R^4}{\lambda\sqrt{\frac{2\tau_p^2 R^{2(p+1)}}{\lambda^{\frac{p+1}{2}}T^2} + \frac{P_R^2 R^4}{\lambda} + \frac{P_\theta^2 R^2}{\lambda} + \frac{P_T^2 T^2 R^2}{2\sqrt{\lambda}}}} = \frac{P_R R^4}{\lambda E} \\
\dot{T} &= \frac{\partial H}{\partial P_T} = \frac{P_T T^2 R^2}{2\sqrt{\lambda}\sqrt{\frac{2\tau_p^2 R^{2(p+1)}}{\lambda^{\frac{p+1}{2}}T^2} + \frac{P_R^2 R^4}{\lambda} + \frac{P_\theta^2 R^2}{\lambda} + \frac{P_T^2 T^2 R^2}{2\sqrt{\lambda}}}} = \frac{P_T T^2 R^2}{2\sqrt{\lambda}E}\,.
\end{aligned} \tag{3.12}$$

Now we use the charge (3.9) to express P_R as function of E and R. Firstly we will presume that various terms in (3.9) contribute in the following way

$$-tE - pTP_T = 0\,, \qquad D = -P_R R\,. \tag{3.13}$$

Then the second equation in (3.13) implies

$$P_R = -\frac{D}{R}\,. \tag{3.14}$$

Inserting this relation into (3.12) we get the differential equation for R

$$\dot{R} = -\frac{D}{\lambda E}R^3 \tag{3.15}$$

that has the general solution

$$\frac{2Dt}{E} = \frac{\lambda}{R^2} - \frac{\lambda}{R_0^2}\,, \tag{3.16}$$

where we have chosen the initial condition that at $t_0 = 0$ the Dp-brane is at the point $\lambda/R_0^2 \gg 1$. The requirement that Dp-brane should move towards to the worldvolume of N D3-branes implies that $D > 0$.

To find the time dependence of T we use (3.13) to express P_T as a function of T and E and insert this expression into the second equation in (3.12) with the result

$$\begin{aligned}
\dot{T} &= -\frac{tET^2R^2}{2pT\sqrt{\lambda}E} = -\frac{tT}{2p\sqrt{\lambda}}\frac{\lambda}{\frac{2Dt}{E} + \frac{\lambda}{R_0^2}} \\
\Rightarrow \qquad \frac{dT}{T} &= -\frac{R_0^2 t}{2p\lambda^{3/2}}\frac{1}{1 + \frac{2DR_0^2}{\lambda E}t} \\
&= \ln\frac{T}{T_0} = -\frac{E}{4p\sqrt{\lambda}}t + \frac{E^2\sqrt{\lambda}}{8pD^2R_0^2}\ln\left(\frac{2R_0^2D}{\lambda E}t + 1\right)\,.
\end{aligned} \tag{3.17}$$

Since $2R_0^2D/\lambda Et \approx \mathcal{O}(1)$ we see that dominant contribution in the regime $T \gg 1$ comes from the term linear in t. Then in the first approximation we obtain the result

$$T = T_0 e^{-\frac{E}{4p\sqrt{\lambda}D}t}\,, \qquad T_0 \gg 1\,. \tag{3.18}$$

We see that the tachyon is lowering its value from the initial large T_0 until it reach the region where the approximation of large tachyon is not valid. On the other hand in the region

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when we can trust this solution we can give the physical interpretation of this solution as follows: It describes the initial stage of the brane creation in which the unstable Dp-brane evolves from the state very close to the vacuum since by presumption $T_0 \ll 1$. At the same time this unstable D-brane moves towards to the worldvolume of D3-branes.

As the second example that can be easily solved we take the ansatz when D splits as

$$D = -pTP_T\,, \qquad tE = -P_R R\,. \tag{3.19}$$

Now the differential equation for R is

$$\dot{R} = -\frac{tR^3}{\lambda} \tag{3.20}$$

that has the solution

$$\frac{\lambda}{R^2} = t^2 + \frac{\lambda}{R_0^2}\,. \tag{3.21}$$

This solution again describes Dp-brane moving towards to the worldvolume of D3-branes. On the other hand the differential equation for T, using (3.19) and also (3.21) takes the form

$$\dot{R} = -\frac{D}{2\sqrt{\lambda}pE}TR^2 \qquad \Rightarrow \qquad \frac{dT}{T} = -\frac{D\sqrt{\lambda}}{2Ep}\frac{1}{t^2 + \frac{\lambda}{R_0^2}} \tag{3.22}$$

with the solution

$$T = T_0 e^{-\frac{DR_0}{2Ep}\arctan\frac{R_0^2 t^2}{\lambda}}\,. \tag{3.23}$$

Since by presumption $\frac{R_0^2}{\lambda} \ll 1$ it is reasonable for $t^2 \ll \frac{\lambda}{R_0^2}$ to approximate $\arctan\frac{R_0^2t^2}{\lambda}$ with $\frac{R_0^2t^2}{\lambda}$. Then (3.23) takes the form

$$T = T_0 e^{-\frac{DR_0^3}{2Ep}t^2}\,. \tag{3.24}$$

Now looking on the time dependence of T given in (3.23) or in its approximate form (3.24) we obtain following physical picture. For $D > 0$ the tachyon is decreasing function so that at same finite time it enters the region where the approximation of the large tachyon breaks down. This time evolution of tachyon describes the initial stage of the process when an unstable D-brane evolves from the state close to its vacuum state to the state that can be interpreted as emergence of the unstable Dp-brane. On the other hand the case when $D < 0$ corresponds to Dp-brane decay in the process of the tachyon condensation.

As the final example we will consider the case when we split D and E as $D = D_1 + D_2, E = E_1 + E_2$ where $D_{1,2}$ and $E_{1,2}$ are related to T, R as

$$D_1 + E_1 t = -pTP_T\,, \qquad D_2 + E_2 t = -P_R R\,. \tag{3.25}$$

Using the second relation in (3.25) we express P_R as a function of R, E_2 and D_2 and insert it into (3.12). Then we obtain the differential equation for R

$$\dot{R} = -\frac{(D_2 + tE_2)R^3}{\lambda E} \tag{3.26}$$

that has the solution

$$\frac{\lambda}{R^2} = \frac{2D_2}{\lambda E}t + \frac{E_2}{E}t^2 + \frac{\lambda}{R_0^2}\,. \tag{3.27}$$

For $D_2 > 0$ the physical interpretation of this solution is the same as in examples studied above. On the other hand for $D_2 < 0$ we obtain following picture: Initially the radial mode grows until it reaches its turning point at $t_* = -\frac{D_2}{E_2}$ where $\dot{R} = 0$. Then Dp-brane starts to move towards to the worldvolume of N D3-branes.

Let us now briefly discuss the time evolution of tachyon. Inserting the first equation in (3.25) into (3.12) we get following differential equation for T

$$\dot{T} = -\frac{(D_1 + E_1 t)TR^2}{2\sqrt{\lambda}pE}\,. \tag{3.28}$$

Inserting (3.27) into the equation given above we obtain the differential equation for T that has roughly following form

$$\frac{dT}{T} = f(t)dt\,, \tag{3.29}$$

where $f(t)$ is a rational function that arises by inserting (3.27) into (3.28). However in order to obtain the rough picture of the time evolution of tachyon it is sufficient to study the equation (3.28) without its explicit integration. For $D_2 > 0$ the tachyon is always decreasing and the tachyon evolution describes the process of the emergence of Dp-brane from the state close to the tachyon vacuum. On the other hand for $D_2 < 0$ we obtain that the tachyon firstly grows until it reaches its turning point at $t_* = D_1/E_1$ when $\dot{T} = 0$ and then again decreases. At any case for nonzero E_1 there is not any possibility to find solution describing the decay of an unstable Dp-brane.

Now we will discuss the dynamics of unstable D0-brane. For $p = 0$ the charge D is equal to

$$D = -tE - P_R R \tag{3.30}$$

that allows us to express P_R as

$$P_R = -\frac{tE + D}{R}\,. \tag{3.31}$$

Then the first equation in (3.12) gives

$$\dot{R} = -R^3\frac{tE + D}{\lambda E} \Rightarrow \frac{\lambda}{R^2} - \frac{\lambda}{R_0^2} = t^2 + \frac{2D}{E}t \tag{3.32}$$

for $t_0 = 0$. We see that an unstable D0-brane is moving towards to the D3-brane worldvolume for $D \leq 0$. On the other hand for $D < 0$ the mode describing the radial position of D0-brane initially grows then it reaches its turning point at $t_* = -\frac{D}{E}$ and then D0-brane moves towards to the stack of D3-branes. In order to obtain relatively simple expressions we will consider the case when D and angular momentum P_θ vanish.

Using the equation (3.32) we can easily determine the time dependence of T. Firstly we express P_T as function of E, T and R

$$P_T^2 = \frac{2\sqrt{\lambda}E^2}{T^2R^2} - \frac{4\tau_p^2}{T^4} - \frac{2t^2E^2}{\sqrt{\lambda}T^2}\,. \tag{3.33}$$

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Then we insert this expression into (3.12) so that we get

$$\dot{T} = \frac{P_T T^2 R^2}{2\sqrt{\lambda}E} = \pm\sqrt{\frac{T^2R^2}{2\sqrt{\lambda}} - \frac{\tau_p^2 R^4}{\lambda E^2} - \frac{t^2T^2R^4}{2\lambda^{3/2}}}\,. \tag{3.34}$$

In fact it is very difficult to solve this equation in the full generality. Then in order to exhibit the main futures of the time evolution of tachyon it is useful to perform some reasonable approximation in (3.34). First of all, let us presume that $\frac{t^2R_0^2}{\lambda} \ll 1 \Rightarrow t^2 \ll \frac{\lambda}{R_0^2}$. Since by presumption $\frac{\lambda}{R_0^2} \gg 1$ we see that this condition is obeyed almost for all t. In this approximation we have $\frac{\lambda}{R^2} \approx \frac{\lambda}{R_0^2}$ so that the equation (3.34) simplifies as

$$\dot{T} = \pm\sqrt{\frac{T^2R_0^2}{2\sqrt{\lambda}} - \frac{\tau_p^2R_0^4}{\lambda E^2}} \tag{3.35}$$

that has the solution

$$T = T_0 \pm \frac{\sqrt{2}\tau_p R_0}{E\lambda^{1/4}}\cosh\left(\frac{\lambda^{1/4}}{\sqrt{2}}\frac{R_0}{\sqrt{\lambda}}t\right). \tag{3.36}$$

As in the case of the unstable D2-brane we obtain two solutions corresponding to D0-brane decay and the initial stage of the emergence of D0-brane respectively. However since the factor in the function $\cosh(x)$ is proportional to $\frac{R_0 t}{\sqrt{\lambda}} \ll 1$ we see that the tachyon is almost constant around the interval under which the approximation used above is valid which implies that the time evolution is very slow.

Let us now determine the time evolution of tachyon for $\frac{t^2R_0^2}{\lambda} \approx 1$. In the first approximation we can write $\frac{\lambda}{R^2} \approx \frac{2\lambda}{R_0^2}$ and hence the differential equation for T takes the form

$$\dot{T} \approx \pm\sqrt{\frac{T^2R_0^2}{8\sqrt{\lambda}} - \frac{\tau_p^2R_0^4}{4\lambda E^2}} \tag{3.37}$$

with the solution

$$T \approx \pm\cosh\left(\frac{R_0}{2\sqrt{2}\sqrt{\lambda}}t\right) \tag{3.38}$$

that again implies that the tachyon is growing or decreasing very slowly.

Let us outline the result that were obtained in this section. We have shown that in order to find additional conserved charge we should restrict to the case of $k = 3$. Then we have studied the dynamics of unstable D2 and D0-branes in the background of N D3-branes. We have seen that the dynamics of the tachyon and radial mode is very reach. In fact, we have argued that the tachyon evolution describes either initial stage of the emergence of unstable Dp-brane or complete decay of this D-brane respectively while it moves towards to the worldvolume of N D3-branes. We have also seen that the form of the time evolution depends on the worldvolume of unstable Dp-brane.

4. Spatial dependent tachyon

In this section we will consider the second example of the tachyon condensation where we will presume that T depends on one spatial coordinate on the non-BPS Dp-brane worldvolume, say $x = \xi^1$, while the modes that describe embedding of the unstable Dp-brane in transverse space are functions of time. From As in the case if the spatial dependent tachyon condensation in flat spacetime one can expect that a codimension one D$(p-1)$-brane will emerge that then moves in the background of N Dk-branes. Let us check that this guess is correct.

As is well known from the study of the tachyon condensation in flat spacetime [32] it is useful to know the form of the stress energy tensor $T_{\mu\nu}$ for the modes living on the worldvolume of non-BPS Dp-brane. In order to find this stress energy tensor it is useful to replace the flat metric $\eta_{\mu\nu}$ that appears in the non-BPS Dp-brane action with the general metric $g_{\mu\nu}$. Then the non-BPS Dp-brane action now takes the form

$$S = -\tau_p \int d^{p+1}\xi \sqrt{-g}\sqrt{F} H_k^{\frac{k-p-4}{4}} \sqrt{\det \mathbf{A}}\,,$$
$$\mathbf{A}^\mu_\nu = \delta^\mu_\nu + H_k g^{\mu\kappa}\partial_\kappa X^m \partial_\nu X^m + F H_k^{\frac{1}{2}} g^{\mu\kappa}\partial_\kappa T \partial_\nu T\,. \tag{4.1}$$

Consequently the stress energy tensor defined as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = -g_{\mu\nu}\mathcal{L} + 2\frac{\delta \mathcal{L}}{\delta g^{\mu\nu}} \tag{4.2}$$

is equal to

$$T_{\mu\nu} = -g_{\mu\nu}\tau_p\sqrt{F} H_k^{\frac{k-p-4}{4}}\sqrt{\det \mathbf{A}} + \tau_p \sqrt{F} H_k^{\frac{k-p-4}{4}} \mathrm{Tr}\frac{\delta \mathbf{A}}{\delta g^{\mu\nu}}\mathbf{A}^{-1}\sqrt{\det \mathbf{A}}\,, \tag{4.3}$$

where

$$\mathrm{Tr}\frac{\delta \mathbf{A}}{\delta g^{xy}}\mathbf{A}^{-1} = \frac{\delta \mathbf{A}^\mu_\kappa}{\delta g^{xy}}(\mathbf{A}^{-1})^\kappa_\mu = \left(H_k \partial_y X^m \partial_\kappa X^m + F H_k^{\frac{1}{2}}\partial_y T \partial_\kappa T\right)(\mathbf{A}^{-1})^\kappa_x\,. \tag{4.4}$$

By definition the stress energy tensor is conserved and hence its components obey following differential equations

$$\partial_\mu \eta^{\mu\kappa} T_{\kappa\nu} = 0\,. \tag{4.5}$$

For the spatial dependent tachyon and the time dependent X^m's the matrix $\mathbf{A}$ is equal to

$$\mathbf{A} = \begin{pmatrix} 1 - H_k \dot{X}^m \dot{X}^m & 0 & 0 \\ 0 & 1 + F H_k^{\frac{1}{2}} T'^2 & 0 \\ 0 & 0 & \mathbf{I}_{(p-1)\times(p-1)} \end{pmatrix} \tag{4.6}$$

so that $\det \mathbf{A} = (1 - H_k \dot{X}^m \dot{X}^m)(1 + H_k^{\frac{1}{2}} F T'^2)$. Then the components of the stress energy tensor are equal to

$$T_{00} = \frac{\tau_p \sqrt{F} H_k^{\frac{k-p-4}{4}} \sqrt{1 + F H_k^{\frac{1}{2}} T'^2}}{\sqrt{1 - H_k \dot{X}^m \dot{X}^m}}\,, \qquad T_{0i} = 0\,, \qquad i = 1, \dots, p$$
$$T_{xx} = -\frac{\tau_p \sqrt{F} H_k^{\frac{k-p-4}{4}} \sqrt{1 - H_k \dot{X}^m \dot{X}^m}}{\sqrt{1 + F H_k^{\frac{1}{2}} T'^2}}\,, \qquad T_{xi} = 0\,, \qquad i = 2, \dots, p$$
$$T_{ij} = -\delta_{ij}\tau_p \sqrt{F} H_k^{\frac{k-p-4}{4}} \sqrt{1 - H_k \dot{X}^m \dot{X}^m}\sqrt{1 + F H_k^{\frac{1}{2}} T'^2}\,. \tag{4.7}$$

In order to find the lower dimensional D-brane we will closely follow [32]. Since for the tachyon kink $T \to -\infty$ for $x \to -\infty$ and $T \to \infty$ for $x \to \infty$ and $\sqrt{F(T)} \to 0$ in this limit we have that $T_{xx} \to 0$ at asymptotic x. Since now T_{xx} is independent on x we see that it must vanish for all x. This however implies that we should have

$$T = \pm\infty \qquad \text{or} \qquad \partial_x T = \pm\infty \qquad (\text{or both}) \qquad \text{for all } x\,. \tag{4.8}$$

Clearly this solution is singular. Following [32] we will show that this solution has finite energy density that is localised on codimension one subspace that is expected to be D$(p-1)$-brane. To see this let us consider the field configuration

$$T(x) = f(ax) \tag{4.9}$$

where $f(u)$ satisfies

$$f(-u) = -f(u)\,, \qquad g'(u) > 0 \forall\, u\,, \qquad g'(u) = \sqrt{F(f(u))}f'(u)\,, \qquad f(\pm\infty) = \pm\infty\,, \tag{4.10}$$

but is otherwise arbitrary function of u. a is constant that we should take to ∞ at the end. In this case we have $T < 0$ for $x < 0$ and $T > 0$ for $x > 0$ and hence this kink is singular as we expected. Note also that the function $g(u)$ is related to $f(u)$ as

$$dg = \frac{1}{\sqrt{1+\frac{f^2}{2}}}df \Rightarrow \sinh\frac{g}{\sqrt{2}} = \frac{f}{\sqrt{2}}\,. \tag{4.11}$$

Then for (4.9) T_{xx} is equal to

$$T_{xx} = -\frac{\tau_p\sqrt{F(f(ax))}H_k^{\frac{k-p-4}{4}}\sqrt{1-H_k\dot{X}^m\dot{X}^m}}{\sqrt{1+a^2F(f(ax))H_k^{\frac{1}{2}}f'^2(ax)}}\,. \tag{4.12}$$

Clearly this component vanishes for all x since the numerator vanishes (except for $x = 0$) and the denominator in the limit $a \to \infty$ is proportional to

$$\sqrt{1+H_kFT'^2} = \sqrt{1+H_ka^2g'^2(ax)} \sim a \to \infty\,. \tag{4.13}$$

One can also easily check, following [32] that the ansatz (4.10) solves the equation of motion for T in the limit $a \to \infty$.

Now using the ansatz (4.10) it is easy to find other components of the stress energy tensor

$$\begin{aligned}
T_{00}(x) &= \frac{\tau_pH_k^{\frac{k-p-4}{4}}}{\sqrt{1-H_k\dot{X}^m\dot{X}^m}}\sqrt{F}\sqrt{1+H_k^{\frac{1}{2}}FT'^2} \\
&= \frac{\tau_pH_k^{\frac{k-p-4}{4}}}{\sqrt{1-H_k\dot{X}^m\dot{X}^m}}H_k^{\frac{1}{4}}F(f(ax))af'(ax)\,, \\
T_{ij}(x) &= -\delta_{ij}\tau_pH_k^{\frac{k-p-4}{2}}\sqrt{1-H_k\dot{X}^m\dot{X}^m}\sqrt{F}\sqrt{1+H_k^{\frac{1}{2}}FT'^2} \\
&= -\delta_{ij}\tau_pH_k^{\frac{k-p-4}{4}}\sqrt{1-H_k\dot{X}^m\dot{X}^m}H_k^{\frac{1}{4}}F(f(ax))af'(ax)
\end{aligned} \tag{4.14}$$

in the limit $a \to \infty$. Then the integrated T_{00} ,T_{ij} associated with the codimension one solution are equal to

$$\begin{aligned}
T_{00}^{kink} &= \int dx T_{00} = \frac{\tau_p H_k^{\frac{k-(p-1)-4}{4}}}{\sqrt{1-H_k \dot{X}^m \dot{X}^m}} \int dx F(f(ax)) a f'(ax) \\
&= \frac{\tau_p H_k^{\frac{k-(p-1)-4}{4}}}{\sqrt{1-H_k \dot{X}^m \dot{X}^m}} \int dy F(y) dy \\
T_{ij}^{kink} &= -\delta_{ij} \tau_p H_k^{\frac{k-(p-1)-4}{2}} \sqrt{1-H_k \dot{X}^m \dot{X}^m} \int dx F(f(ax)) a f'(ax) \\
&= -\delta_{ij} \tau_p H_k^{\frac{k-(p-1)-4}{2}} \sqrt{1-H_k \dot{X}^m \dot{X}^m} \int dy F(y) dy \,, \qquad (4.15)
\end{aligned}$$

where $y = f(ax)$. Thus $T_{\alpha\beta}^{kink}$,$\alpha, \beta = 0, 2, \dots, p$ depend on F and not on the form of $f(u)$. It is clear from the form of the function F that most of the contribution is contained in the finite range of y. In fact, in the limit $a \to \infty$ the stress energy tensor $T_{\alpha\beta}^{kink}$ is localised on codimension one D$(p-1)$-brane with the tension given as

$$T_{p-1} = \tau_p \int dy F = \tau_p \int \frac{dy}{1+\frac{y^2}{2}} = \sqrt{2}\pi\tau_p = \frac{(2\pi)}{(2\pi)^{p+1}} = \frac{1}{(2\pi)^p} \qquad (4.16)$$

using the fact that τ_p is equal to

$$\tau_p = \frac{\sqrt{2}}{(2\pi)^{p+1}} \,. \qquad (4.17)$$

In other words, the spatial dependent condensation leads to the emergence of D$(p-1)$-brane that is moving in the Dk-brane background. Since this situation was analysed previously in [31, 16] we will not repeat it there. In fact, the aim this section was to show that one can construct codimension one objects on the worldvolume of a non-BPS Dp-brane even in the nontrivial background produced by stack of N Dk-branes.[6]

5. Conclusion

This paper was devoted to the study of the dynamics of a non-BPS Dp-brane in the background of N Dk-branes, where $k > p$. It can be also considered as an extension of the previous paper [11] where the dynamics of a non-BPS Dp-brane in the background of N NS5-branes was studied. We were mainly interested in the time evolution of the tachyon and radion mode on the worldvolume of the non-BPS Dp-brane. We have shown that the main properties of the dynamics of the unstable Dp-brane strongly depends on dimensions of the background Dk-branes and the probe Dp-brane. Generally the time dependent tachyon evolution either reduces or increases the initial tension of a non-BPS Dp-brane

[6]Before we conclude this section we should say few words about the fate of the WZ term in the non-BPS Dp-brane action in case of spatial dependent tachyon. In this case this term is approximately equal to $\sim \int T' dx \wedge C_{p-1}(X^m)$. Its contribution to the tachyon equation of motion is proportional to $\int dx \frac{d\delta T}{dx} C \Rightarrow -\int dx \delta T \frac{d}{dx} C(X^m) = 0$ since by presumption X^m are not functions of x.

and hence one can expect that the tachyon condensation would have profound impact on the motion of an unstable Dp-brane in the curved spacetime. To support this claim we have restricted ourselves to the study of the unstable D-brane dynamics in the region of the field theory space where T is large and $\frac{\lambda}{R^{7-k}} \gg 1$. We have then argued that for the background of N D3-branes a new symmetry on the worldvolume of the unstable Dp-brane emerges. Using now the conserved charge that is generator of this symmetry we were able to sketch the rough picture of the evolution of T and R that live on the worldvolume of non-BPS D2 and D0-branes.

We mean that the fact that the form of the tachyon condensation strongly depends on dimensions of background Dk-branes and a probe Dp-brane is related to the fact the charge D is defined for the background of N D3-branes only. It would be certainly very interesting to find general form of the transformation of the worldvolume fields that will leave the effective action invariant. We hope that the possible existence of this symmetry could be helpful for better understanding of the geometrical origin of the tachyon conjecture. We mean that this symmetry could be related to the generalised symmetry discussed in [33, 34]. We plan to return to this problem in the future work.

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